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Principal Component and Multiple Regression Analysis for Steel Fiber Reinforced Concrete (SFRC) Beams

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Abstract: This study evaluates the shear strength of steel fiber reinforced concrete (SFRC) beams from a database, which consists of extensive experimental results of 222 SFRC beams having no stirrups. In order to predict the analytical shear strength of the SFRC beams more precisely, the selected beams were sorted into six different groups based on their ultimate concrete strength (low strength with $f_c' < 50$ MPa and high strength with $f_c' < 50$ MPa), span-depth ratio (shallow beam with $a/d \geq 2.5$ and deep beam with $a/d < 2.5$) and steel fiber shape (plain, crimped and hooked). Principal component and multiple regression analyses were performed to determine the most feasible model in predicting the shear strength of SFRC beams. A variety of statistical analyses were conducted, and compared with those of the existing equations in estimating the shear strength of SFRC beams. The results showed that the recommended empirical equations were best suited to assess the shear strength of SFRC beams more accurately as compared to those obtained by the previously developed models.

Keywords: shear strength, steel fiber, reinforced concrete beams, statistical analysis, principal component regression.

List of Symbols

a	Shear span, mm
a/d	The ratio of shear span and effective depth
A_{sw}	Cross-sectional area of shear reinforcement, mm^2
b	Beam width, mm
d	Effective beam depth, mm
d_a	Maximum aggregate size, mm
d_f	Fiber diameter, mm
F	Fiber factor
F_y	Yield strength of shear reinforcement, MPa
f_c'	Concrete compressive strength, MPa
f_f	Flexural strength of plain concrete, MPa
f_{sp}	Splitting strength of plain concrete, MPa
h	Beam height, mm
l_f	Fiber length, mm
l_f/d_f	Fiber aspect ratio
η_o	Fiber orientation factor
ρ	Tensile reinforcement ratio, %
SFRC	Steel fiber reinforced concrete
τ	Fiber-matrix interfacial bond strength, MPa
V/M	External shear to moment ratio
V_c	Shear resistance of the concrete, MPa
v_f	Fiber volume in percentage
V_{EXP}	Experimental shear strength, MPa
V_{ANA}	Predicted shear strength, MPa
V_u	Ultimate shear strength, MPa

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1. Introduction

The concept of fiber reinforcing in concrete was introduced more than a century ago, and since then, various types of fibers have been utilized in concrete. Steel fibers were first utilized as effective concrete reinforcements in 1960, and they have been used extensively in many applications of large industrial floors, bridge deck overlays, airport runways, pavements, spillways, dams, slope stabilizations, and many precast products (Dinh 2007, 2010). The steel fibers form bridges through developing cracks in the concrete that provide more resistance against crack growth (Narayanan and Darwish 1987; Li et al. 1992; Lim and Oh 1999). Thus, the inclusion of steel fibers to an ordinary reinforced concrete beam suppresses shear failure in favour of more ductile behaviour (Mansur et al. 1986; Narayanan and Darwish 1987; Ramakrishna and Sundararajan 2005). The steel fibers can prevent excessive diagonal tensile cracking and localization of the tensile crack damage (Choi et al. 2007). A number of existing studies have clearly established the potential use of fiber reinforcement for enhancing the shear capacity of reinforced concrete beams (Batson et al. 1972; Swamy and Bahia 1985; Li et al. 1992; Khuntia et al. 1999; Dupont and Vandewalle 2003; Kang et al. 2011). The increase in shear strength contributed by the steel fibers widely varied from 13 to 170 % (Narayanan and Darwish 1988), 58–125 % (Greenough and Nehdi 2008) and 22–89 % (Swamy et al. 1993) due to the variations of fiber volume, its aspect ratio and anchorage condition, tensile reinforcement ratio, and compressive strength of concrete beams, respectively.

Tables 1 and 2 shows the empirical models and constrains in calculating the shear strength of SFRC beams. As can be noted, the shear behavior of SFRC beams mainly depends on concrete compressive strength (f_c'), tensile reinforcement ratio (ρ), span-depth ratio (a/d), fiber aspect ratio (l_f/l_d) and the amount of fiber in concrete (v_f). The ultimate shear strength of SFRC beams decreases with an increase in the span-depth ratio of beam (Narayanan and Darwish 1987; Mansur et al. 1986; Ashour et al. 1992; Li et al. 1992; Imam et al. 1994; Noghabai 2000; Dinh 2007) and increases with increasing flexural reinforcement ratio (Narayanan and Darwish 1987; Imam et al. 1994; Dinh 2007) and ultimate compressive strength (Narayanan and Darwish 1987; Kwak el al. 2002; Dinh 2007). The shear strength also depends on the amount/volume of steel fibers in the concrete mixture (Khaloo and Kim 1997; Dinh 2007; Madan et al. 2007; Yakoub 2011). Additionally, the aspect ratio and anchorage conditions of the steel fibers greatly influence the shear strength of the SFRC beams (Narayanan and Darwish 1987; Li et al. 1992; Khaloo and Kim 1997).

The shear strength prediction models suggested by the previous research studies were mostly complex and confined to nonlinear regression equations. The past investigations were lacked of simple models, whose preferences are not just for philosophical but also for practical aspects. Multiple linear regression model is a typically form of simple models where more than one independent variables are present. The principal component regression (PCR), special types of regression, can also be utilized on the dataset of multiple linear regression. The PCR uses a set of values of linearly uncorrelated variables called principal components (PCs). The principal component analysis (PCA) compresses and classifies data by evaluating a new set of variables, smaller than the original set of variables, which holds most information of the dataset. PCA determines a set of orthogonal vectors, called loading vectors, which can be ordered by the amount of variance explained in the loading vector directions.

A multiple linear regression model is shown in Eq. (1), and its least-squares solution is given by Eq. (2), where X^TX is singular because of the variables in X exceeds the number of observations or the collinearities. In order to elude the singularity of X^TX, the principal component regression (PCR) decomposes X into orthogonal scores T and loadings P, as shown in Eq. (3). As such, regressing Y does not only depend on X itself but also the first a columns of scores T. In the principal component regression, the scores are present by the left singular vector of X multiplied with the corresponding singular values, while the loadings are shown the right singular vectors of X. In PCR, the X matrix consists of the first a principal components (PCs), usually obtained from the singular value decomposition (SVD). Equation (4) presents the X in terms of scores T and loadings P. Finally, and Eq. (5) shows the regression coefficients of the scores.

$$Y = XB + E \tag{1}$$

$$B = \left(X^TX\right)^{-1}X^TY \tag{2}$$

$$X = TP \tag{3}$$

$$X = X_{(a)} + \varepsilon_x = \left(U_{(a)}D_{(a)}\right)V_{(a)}^T + \varepsilon_x = T_{(a)}P_{(a)}^T + \varepsilon_x \tag{4}$$

$$B = P\left(T^TT\right)^{-1}T^TY = V\,D^{-1}U^TY \tag{5}$$

where the subscript a has been dropped for clarity.

A general agreement existed among the previous research studies that a substantial gain in shear strength of SFRC beams is experienced due to the increase in compressive strength, tensile reinforcement ratio, and fiber volume, and due to the decrease in the span-depth ratio of beam. The existing models in predicting shear capacity of SFRC beams often generated results those are typically in excess of or smaller than the experimental values. As such, an accurate model is extensively needed for the shear strength prediction of SFRC beams. The purpose of this study is to develop empirical models for predicting the shear capacity of SFRC beams using the existing experimental results.

2. Research Significance

The previous investigations in assessing the shear strength of SFRC beams were mostly confined to a limited number of beams having a narrow range of ultimate compressive strength, span-depth ratio of beam, tensile reinforcement ratio, fiber aspect ratio and the volume of steel fibers. Additionally, the existing models were mostly unable to accurately predict the shear strength of SFRC beams. Therefore, the development of an accurate model/equation for predicting the shear strength of SFRC beam is needed for the development of valid design codes, which will enable SFRC to be used as a common building material. This study enhances the existing research studies in introducing the multiple regression models and the principal component regression models in predicting the shear capacity of SFRC beams.

3. Existing Experimental Results

A database containing 222 SFRC beams without stirrups was compiled from 23 existing experimental studies conducted by Batson et al. (1972), Swamy and Bahia (1985), Mansur et al. (1986), Sharma (1986), Murty and Venkatacharyulu (1987), Narayanan and Darwish (1987), Ashour et al. (1992), Li et al. (1992), Swamy et al. (1993), Tan et al. (1993), Imam et al. (1994), Shin et al. (1994), Khaloo and Kim (1997), Lim and Oh (1999), Noghabai (2000), Kwak

Table 1 Existing models for shear strength prediction of SFRC shallow beams without stirrups.

Study	Proposed v_u equation[a]	Equation description	Limitations
Greenough and Nehdi (2008)	$v_u = 0.35 * \left(1 + \sqrt{\frac{400}{\mathrm{d}}}\right) * (f_c')^{0.18} * \left((1+F) * \rho * \frac{d}{a}\right)^{0.4} + 0.9 * \eta_o * \tau * F$	Genetic algorithm	$a/d > 2.5$ $f_c' < 70$ MPa
Mansur et al. (1986)	$v_u = v_c + \sigma_{tu} * b * d$, where, $v_c = \left(0.16 * \sqrt{f_c'} + 17.2 * \frac{\rho V d}{M}\right)$	ACI code modification	$V_c < (0.29 (f_c')^{0.5}) bd$
Li et al. (1992)	$v_u = 1.25 + 4.68 * \left((f_t f_{sp})^{\frac{3}{4}} + \left(\rho * \frac{d}{a}\right)^{\frac{1}{3}} * (d)^{-\frac{1}{3}}\right)$	Regression analysis	$a/d \geq 2.5$
Khuntia et al.(1999)	$v_u = (0.167 + .25 * \mathrm{F}) * \sqrt{\mathrm{f}_c'}$	ACI code modification	$a/d \geq 2.5$ $0.25 \leq \rho \leq 2$ $20 \leq f_c' \leq 100$ MPa
Ashour et al. (1992)	$v_u = \left(2.11 * \sqrt[3]{\mathrm{f}_c'} + 7 * F\right) * \left(\rho * \frac{\mathrm{a}}{\mathrm{d}}\right)^{0.3333}$	Regression analysis	$a/d > 2.5$
Imam et al. (1994)	$v_u = 0.7 * \left(\frac{1}{\sqrt{1 + \frac{d}{25 d_a}}}\right) * \sqrt[3]{\rho} * \left(f_c'^{0.44}(1 + F^{0.33}) + 870 * \sqrt{\frac{\rho}{\left(\frac{a}{d}\right)^5}}\right)$	Regression analysis	Maximum aggregate size
Kwak et al. (2002)	$v_u = 3.7 * \left(\left(\frac{f_c'}{20 - \sqrt{F}} + 0.7 + \sqrt{F}\right)^{\frac{2}{3}} * \left(\frac{\rho d}{a}\right)^{\frac{1}{3}}\right) + 0.8 * (0.41 * \tau * F)$	–	$a/d > 3.4$
Khaloo and Kim (1997)	$v_u = \left(0.65 + 0.123 * V_f + 0.080 * (V_f)^2 - 0.013 * (V_f)^3\right) * \sqrt{\mathrm{f}_c'}$	Regression analysis	$l/d = 29$
	$v_u = 0.65 + 0.46 * V_f - 0.080 * (V_f)^2 * \sqrt{\mathrm{f}_c'}$	Regression analysis	$l/d = 58$
Shin et al. (1994)	$v_u = 0.19 * f_{sp} + 93 * \rho * \left(\frac{d}{a}\right) + 0.834 * (0.41 * \tau * F)$	–	$a/d \geq 3$ HSC
Sharma (1986)	$v_u = \frac{2}{3} * \mathrm{f_t} * \left(\frac{d}{a}\right)^{\frac{1}{4}}$ where, $f_t \approx 9.5 * (\sqrt{f_c'}), psi$	–	–
Narayanan and Darwish (1988)	$v_u = \left(0.24 * \left(\frac{f_c'}{20 - \sqrt{F}} + 0.7 + \sqrt{F}\right) + 80 * \rho * \frac{d}{a}\right) + 0.41 * \tau * F$	Regression analysis	$a/d > 2.8$

[a] $F = V_f \frac{l_f}{d_f} D_f$; $\alpha = 1 \frac{N}{\mathrm{mm}^2}$; $\alpha = 1 \frac{N}{\mathrm{mm}^2}$; $\tau = 4.15 \frac{N}{\mathrm{mm}^2}$.

Table 2 Existing models for shear strength prediction of SFRC deep beams without stirrups.

Study	Proposed v_u equation[a]	Equation description	Limitations
Mansur et al. (1986)	$v_u = v_c + \sigma_{tu} * b * d$, where, $v_c = \left(0.16 * \sqrt{f'_c} + 17.2 * \frac{\rho V d}{M}\right)$	ACI code modification	$V_c < (0.29(f'_c)^{0.5})bd$
Li et al. (1992)	$v_u = 9.16 * \left((f_t)^{\frac{2}{3}} * (\rho)^{\frac{1}{3}} * \left(\frac{d}{a}\right)\right)$	Regression analysis	$a/d \leq 2.5$
Khuntia et al. (1999)	$v_u = \left(0.167 * \left(2.5 * \frac{d}{a}\right) + .25 * F\right) * \sqrt{f'_c}$	ACI code modification	$a/d \leq 2.5$ $0.25 \leq \rho \leq 2$ $20 \leq f'_c \leq 100$ MPa
Ashour et al. (1992)	$v_u = \left(\frac{2.5}{\frac{a}{d}}\right) + 0.41 * \tau * F * \left(2.5 - \frac{a}{d}\right)$	Regression analysis	$a/d < 2.5$
	$v_u = \left(0.7 * \sqrt{f'_c} + 7F\right) * \frac{d}{a} + 17.2\rho * \frac{d}{a}$	ACI code modification	More accurate for HSC beams
Imam et al. (1994)	$v_u = 0.7 * \left(\frac{1}{\sqrt{1 + \frac{d}{25 d_a}}}\right) * \sqrt[3]{\rho} * \left(f_c^{i0.44}(1 + F^{0.33}) + 870 * \sqrt{\frac{\rho}{\left(\frac{a}{d}\right)^5}}\right)$	Regression analysis	Maximum aggregate size
Kwak et al. (2002)	$v_u = 3.7 * \left(3.4 * \left(\frac{d}{a}\right) * \left(\frac{f'_c}{20 - \sqrt{F}} + 0.7 + \sqrt{F}\right)^{\frac{2}{3}} * \left(\frac{\rho d}{a}\right)^{\frac{1}{3}}\right) + 0.8 * (0.41 * \tau * F)$	–	$a/d \leq 3.4$
Khaloo and Kim (1997)	$v_u = \left(0.65 + 0.123 * V_f + 0.080 * (V_f)^2 - 0.013 * (V_f)^3\right) * \sqrt{f'_c}$	Regression analysis	$l/d = 29$
	$v_u = 0.65 + 0.46 * V_f - 0.080 * (V_f)^2 * \sqrt{f'_c}$	Regression analysis	$l/d = 58$
Shin et al. (1994)	$v_u = 0.22 * f_{sp} + 217 * \rho * \left(\frac{d}{a}\right) + 0.834 * (0.41 * \tau * F)$	–	HSC $a/d < 3$
Sharma (1986)	$v_u = \frac{2}{3} * f_t * \left(\frac{d}{a}\right)^{\frac{1}{4}}$ where, $f_t \approx 9.5 * (\sqrt{f'_c}), psi$	–	–
Narayanan and Darwish (1988)	$v_u = 2.8 * \frac{d}{a} * \left(0.24 * \left(\frac{f'_c}{20 - \sqrt{F}} + 0.7 + \sqrt{F}\right) + 80 * \rho * \frac{d}{a}\right) + 0.41 * \tau * F$	Regression analysis	$a/d \leq 2.8$

[a] $F = V_f \frac{l_f}{d_f} D_f$; $\alpha = 1 \frac{N}{mm^2}$; $\alpha = 1 \frac{N}{mm^2}$; $\tau = 4.15 \frac{N}{mm^2}$.

et al. (2002), Cho and Kim 2003, Rosenbusch and Teutsch (2003), Cucchiara et al. (2004), ACI 318R-05 (2005), Adhikary and Mutsuyoshi (2006), Minelli and Vecchio (2006), Greenough and Nehdi (2008), and Dinh et al. (2010). The experimental data of the selected SFRC beams were divided into two groups of shallow and deep beams based on the beam span-depth ratio of greater than or equal to 2.5 and that of less than 2.5, respectively (Choi and Park 2007). The shallow and deep beam groups were also sorted based on their ultimate compressive strength of lower than 50 MPa (low strength concrete) and that of greater than or equal to 50 MPa (high strength concrete) (Esfahani and Rangan 1998; ACI-318R-05 2005), and the anchorage of steel fibers (hooked or plain and crimped or the combination of plain, crimped and hooked). As a result, the selected SFRC beams were grouped into six categories according to their ultimate compressive strength, span-depth ratio and the shape of steel fibers. They are: (a) low strength beams with $a/d < 2.5$ and hooked fibers (*LS-DB-H*), (b) low strength beams with $a/d < 2.5$ and plain and crimped fibers (*LS-DB-PC*), (c) High strength beams with $a/d < 2.5$ and plain, crimped and hooked fibers (*HS-DB-PCH*), (d) High strength SFRC beams with $a/d \geq 2.5$ and plain, crimped and hooked fibers (*HS-SB-PCH*), (e) Low strength SFRC beams with $a/d \geq 2.5$ and hooked fibers (*LS-SB-H*), and (f) Low strength beams with $a/d \geq 2.5$ and plain/crimped fibers (*LS-SB-PC*). The number of SFRC beams in various configurations incorporated in this study is 41, 58, 31, 49, 33 and 10 for the *LS-DB-H*, *LS-DB-PC*, *HS-DB-PCH*, *HS-SB-PCH*, *LS-SB-H* and *LS-SB-PC*, respectively. The detailed parameters of the SFRC beams are shown in Table 3.

4. Results and Discussions

Figure 1 shows the relationship among the independent variables of compressive strength (CS), reinforcement ratio (RR), span-depth ratio (SDR), aspect ratio (AR), fiber volume (FV) and fiber type (FT) and the response variable of shear strength (SS). As can be shown in Fig. 1, the ultimate shear strength of SFRC beams increased with an increase in the ultimate compressive strength, decreased with an increase in span-depth ratio, and was inconsistent with the reinforcement ratio and fiber aspect ratio. Furthermore, no particular trend was observed between the strength of SFRC beams and the aspect ratio of steel fibers used in concrete.

4.1 Proposed Shear Strength Predict Model

4.1.1 Multiple Regression Analysis

In order to predict the analytical shear strength of SFRC beams more accurately, the experimental shear strength of the selected six types of SFRC beams was analyzed with the five parameters of ultimate compressive strength (f_c'), tensile reinforcement ratio (ρ), span-depth ratio (a/d), shape factor of steel fiber (l_f/d_f) and the volume of steel fibers (v_f). A general pattern of multiple regression analysis, as shown in Eq. 6, was conducted in optimizing the regression parameters of a, b, c, d, e and f. The interactions among the parameters had been neglected as suggested by other researchers (Sharma 1986; Narayanan and Darwish 1988; Li et al. 1992; Khuntia et al. 1999; Kwak et al. 2002; Greenough and Nehdi 2008).

$$Y = aX_1 + bX_2 + cX_3 + dX_4 + eX_5 + f \qquad (6)$$

Here, Y is the shear strength of SFRC beams, X_1 is the ultimate compressive strength, X_2 is the tensile reinforcement ratio, X_3 is the span-depth ratio, X_4 is the shape factor of steel fiber, X_5 is the volume of steel fibers expressed as a percentage of the total volume of concrete, and a, b, c, d, e and f are the respective coefficients.

Several statistical analyses, such as the performance factor ($PF = V_{EXP}/V_{ANA}$), coefficient of variation (COV), coefficient of determination (R^2), average absolute error (AAE), and Chi squared value (λ^2), were conducted for the proposed simplified and nonlinear models. Additionally, the mean, standard deviation (SD), coefficient of variation (COE) and confidence at the significance level (α) of 0.05 for the performance factor were evaluated. The above mentioned statistical parameters were also compared with those generated by the previously developed equations in predicting the shear strength of SFRC beams as presented in Tables 1 and 2 except for the models that consisted of the shear and moment of plain concrete beams (Mansur et al. 1986), and the beams that were too specific and designed only for two fiber aspect ratios of 29 and 58 (Khaloo and Kim 1997).

The proposed models in predicting the shear strength of the *LS-DB-H*, *LS-SB-H*, *LS-SB-PC*, *HS-SB-PCH*, *LS-SB-H* and *LS-SB-PC* beams are shown in Eqs. 7–12, respectively.

Table 3 Database details for the selected SFRC beams.

Beam ID	f_c' (MPa)	ρ (%)	a/d	l_f/d_f	v_f (%)
HS-SB-PCH	50.8–111.5	1.5–5.72	2.5–6.0	55–133	0.25–3.0
LS-SB-H	22.7–49.2	1.1–3.10	2.5–4.0	50–80	0.25–1.5
LS-SB-PC	32.1–49.8	1.22–5.72	2.5–4.91	42.8–133	0.22–1.5
HS-SB-PCH	22.7–48.7	0.9–3.89	0.8–2.0	29.1–100	0.25–2.0
LS-SB-H	28.7–47.2	1.22–4.31	0.7–2.0	50–100	0.25–1.0
LS-SB-PC	50.8–111.5	1.5–5.72	2.5–6.0	55–133	0.25–3.0

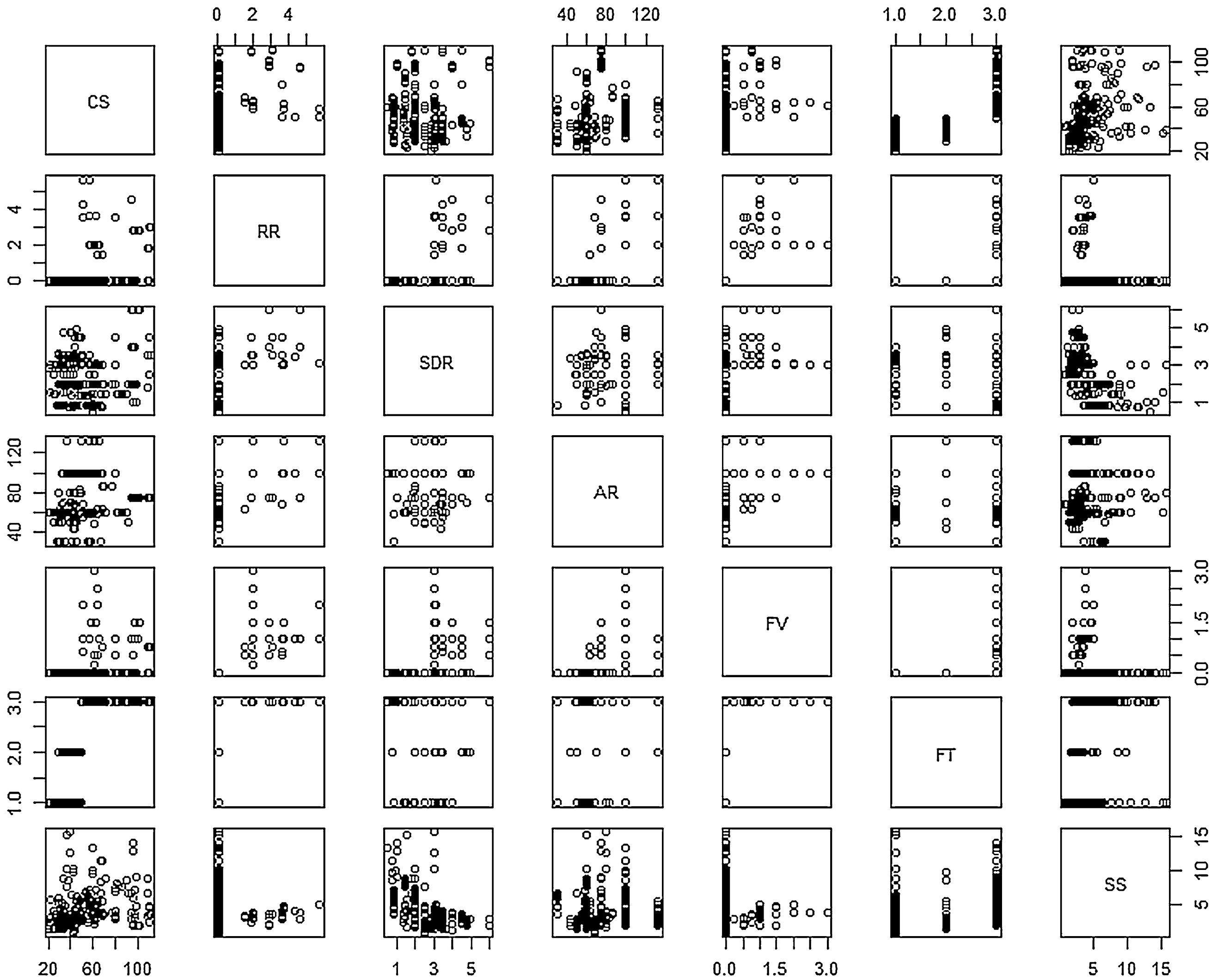

Fig. 1 The relationship among the independent variables of compressive strength (CS), reinforcement ratio (RR), span-depth ratio (SDR), aspect ratio (AR), fiber volume (FV) and fiber type (FT) and the response variable of shear strength (SS).

The analytical shear strength of the above mentioned SFRC beams was also determined and compared with that obtained by the existing models. The statistical analyses for the existing and proposed models for shear capacity of *LS-DB-H*, *LS-SB-H*, *LS-SB-PC*, *HS-SB-PCH*, *LS-SB-H* and *LS-SB-PC* beams are presented in Tables 4–9, respectively. The experimental and analytical shear strength values for all above mentioned SFRC beams were correlated, and the results are shown in Fig. 2.

$$V_u = 0.034 f_c' + 201.745\,\rho - 1.811\,(a/d) + 0.077\,\left(l_f/d_f\right) + 150.35\,v_f - 2.707 \tag{7}$$

$$V_u = 0.140 f_c' + 32.292\,\rho - 3.543\,(a/d) - 0.002\,\left(l_f/d_f\right) + 142.663\,v_f + 3.590 \tag{8}$$

$$V_u = 0.087 f_c' + 66.323\,\rho - 4.407\,(a/d) + 0.035\,\left(l_f/d_f\right) + 99.107\,v_f + 1.787 \tag{9}$$

$$V_u = 0.010 f_c' + 0.238\,\rho - 0.732\,(a/d) + 0.0031\,\left(l_f/d_f\right) + 0.1304\,v_f + 4.332 \tag{10}$$

$$V_u = 0.034 f_c' + 201.745\,\rho - 1.811\,(a/d) + 0.077\,\left(l_f/d_f\right) + 150.35\,v_f - 2.707 \tag{11}$$

$$V_u = 0.050 f_c' + 12.788\,\rho - 0.081\,(a/d) + 0.0011\,\left(l_f/d_f\right) + 84.804\,v_f - 0.287 \tag{12}$$

For the beams containing hooked fibers and having low compressive strength with $a/d < 2.5$, Table 4 illustrates that the previously developed models generated a wide range of low R^2 values (0.00–0.10) and high Chi squared values (39.16–168.45). Additionally, the mean of the performance factor differed from 0.55 to 1.82, the standard deviation of *PF*s fluctuated from 0.35 to 1.31, and *COV* of *PF*s varied from 50.07 to 72.04. The existing models generated a large amount of scattered data, which predicted an under or overestimation of shear capacity. On the other hand, the proposed model outperformed all previous models with a higher R^2 value of 0.72, a lower λ^2 factor of 12.10, and the mean of *PFs* of 1.04, the standard deviation of *PF*s of 0.33, and the *COV* of 31.68. The model also produced the least amount of scattered data (Fig. 2a), which indicated less error in the analytical results as compared to those obtained by the

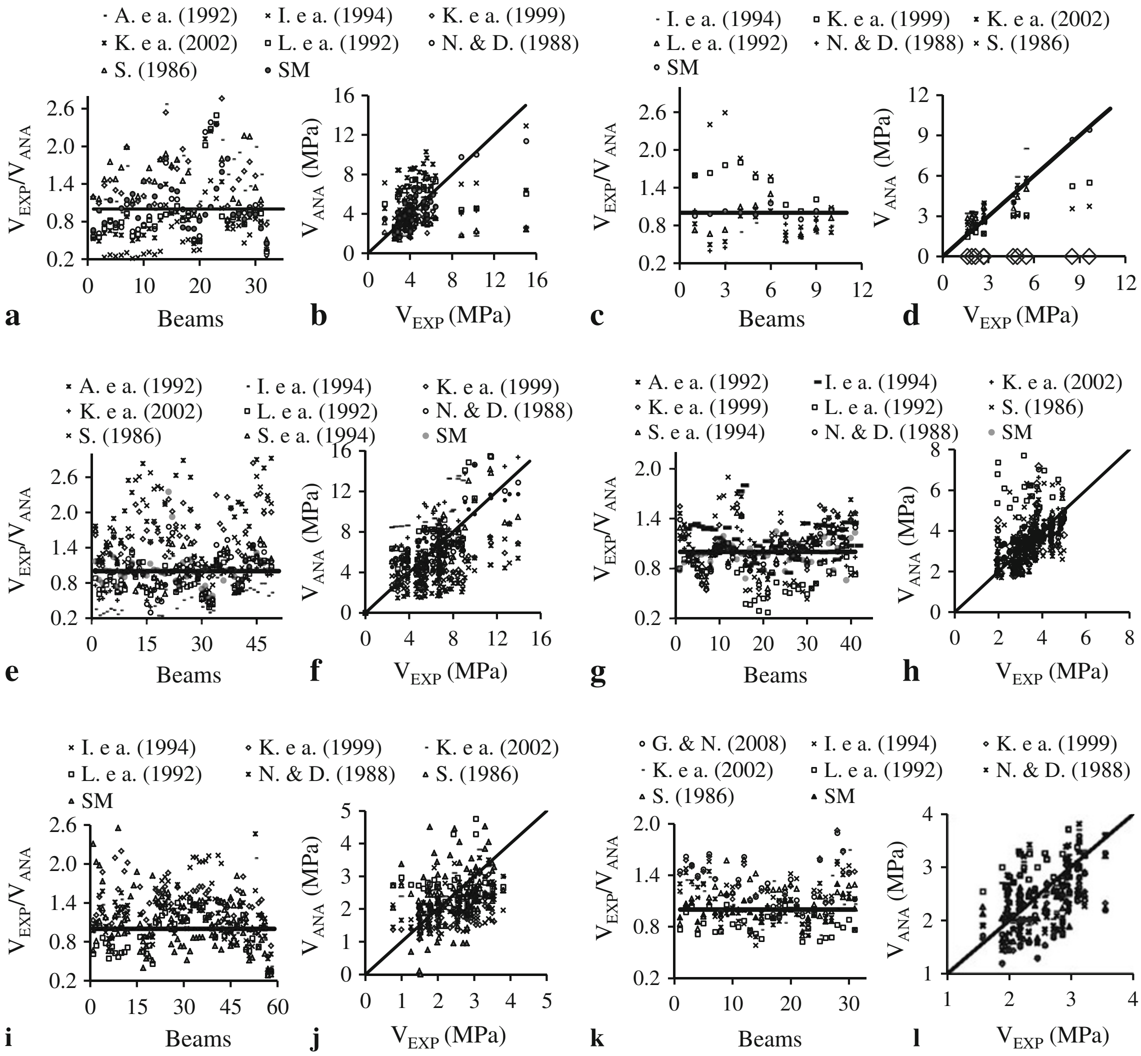

Fig. 2 Comparison between the predicted and experimental shear strength of SFRC beams.

existing models for generating the shear strength of *LS-DB-H* beams.

For the low strength SFRC beams with $a/d < 2.5$ and plain/crimped fibers (LS-DB-PC), Table 5 demonstrates that the model suggested by Khuntia et al. (1999) produced the highest R^2 value of 0.96, and the equation proposed by Li et al. (1992) yielded *AAE* of 33.35, λ^2 of 4.13, and mean of V_{EXP}/V_{ANA} of 0.85. The proposed model generated R^2 values of 0.99, the *AAE* values of 5.09, and λ^2 values of 0.15, respectively. Additionally, the suggested equation produced the mean of *PFs* of 1.00 with the standard deviations of 0.07, and *COVs* of 7.02. Moreover, the proposed models produced the narrowest dispersion (Fig. 2b), which indicated more precision in determining the shear strength of the selected SFRC beams. Thus, the proposed model was best suited to determine the analytical shear strength of SFRC beams as compared to that obtained by the previously developed models.

Table 6 shows that the existing models in shear strength prediction for the high strength beams with $a/d < 2.5$ and plain, crimped and hooked fibers (HS-DB-PCH) generated a wide range of R^2 (0.29–0.69), *AAE* (30.38–122.16), λ^2 (29.50–328.36) and the standard deviation of *PF*s (0.63–2.40). The proposed equation yielded R^2 values of 0.77, λ^2 value 15.67, and *AAE* value of 18.12. Additionally, the suggested model showed the mean of *PFs* of 1.02 with the lowest standard deviations of the *PFs* of 0.29, and *COVs* of the *PFs* of 28.79, and the confidences of 0.08 at the 95 % significance level. The results of the study indicated that the existing equations were not able to better predict the shear strength of *HS-DB-PCH* beams, whereas the proposed empirical model superseded all the existing models in evaluating the shear strength of SFRC beams.

Among all the models in predicting the shear capacity of high strength SFRC beams with $a/d \geq 2.5$ and plain, crimped and hooked fibers (HS-SB-PCH), as shown in Table 7, the equations proposed by Li et al. (1992) was most inaccurate, and those recommended by Ashour et al. (1992) underestimated the shear capacity of *HS-SB-PCH* beams. Of the existing models, the procedure suggested by Kwak et al. (2002) showed the best results with an R^2 value of 0.64, λ^2 of 17.43 and *AAE* of 15.57. Based on the findings from Table 7 and Fig. 2g and h, the proposed model generated the most precise results compared to all existing models.

Table 4 Statistical analysis of shear strength prediction models for low strength SFRC beams with *a/d* < 2.5 and hooked fibers (*LS-DB-H*).

Study	V_{EXP}/V_{ANA}				*AAE*	λ^2	R^2	Rating[b] (1–8)
	Mean	*SD*	*COV* (%)	Confidence[a]				
Ashour et al. (1992)	1.82	1.31	72.04	0.46	32.1	154.2	0.01	8
Imam et al. (1994)	0.55	0.35	64.79	0.12	169.7	168.5	0.04	5
Khuntia et al. (1999)	1.80	1.19	65.87	0.41	32.8	145.0	0.01	6
Kwak et al. (2002)	0.87	0.51	58.25	0.18	55.5	50.6	0.04	4
Li et al. (1992)	0.91	0.48	52.20	0.17	40.7	40.1	0.06	3
Narayanan and Darwish (1988)	1.02	0.51	50.07	0.18	35.2	39.2	0.10	2
Sharma (1986)	1.79	1.20	66.73	0.41	37.4	147.0	0.00	7
Proposed model	1.04	0.33	31.68	0.11	26.2	12.1	0.72	1

[a] $\alpha = 0.05$, [b] 1(excellent) and 7(worst).

Table 5 Statistical analysis of shear strength prediction models for low strength SFRC beams with *a/d* < 2.5 and plain and crimped fibers (*LS-DB-PC*).

Study	V_{EXP}/V_{ANA}				*AAE*	λ^2	R^2	Rating[b] (1–7)
	Mean	*SD*	*COV* (%)	Confidence[a]				
Imam et al. (1994)	0.63	0.29	46.18	0.18	121.6	65.4	0.82	7
Khuntia et al. (1999)	1.43	0.30	20.79	0.18	26.9	9.8	0.96	2
Kwak et al. (2002)	0.74	0.18	24.47	0.11	42.5	9.8	0.89	3
Li et al. (1992)	0.85	0.24	28.19	0.15	33.4	4.1	0.88	4
Narayanan and Darwish (1988)	0.83	0.26	30.87	0.16	40.1	15.1	0.86	5
Sharma (1986)	1.51	0.66	43.36	0.41	36.9	21.8	0.79	6
Proposed Model	1.00	0.07	7.02	0.04	5.1	0.2	0.99	1

[a] $\alpha = 0.05$, [b] 1(excellent) and 7(worst).

Based on the statistical significance of the shear strength prediction models for the low compressive strength SFRC beams with $a/d \geq 2.5$ and hooked fibers (LS-SB-H), as shown in Table 8, the most overestimated mean of performance factor (1.68) and the highest standard deviation (1.03) were generated by Khuntia et al. (1999). The highest R^2 value from the existing models was shown to be 0.42 (Kwak et al. 2002), whereas the proposed model generated an R^2 value of 0.82. Figure 2i and j demonstrate that the proposed model outperformed the existing models in estimating the shear strength of *LS-SB-H* beams.

As can be seen in Table 9, the existing models in predicting the shear strength of SFRC beams with $a/d \geq 2.5$ and plain/crimped fibers (LS-SB-PC) covered a relatively wider range of R^2 (0.12–0.43), *AAE* (17.62–174.25), Chi squared (3.96–51.04) and the mean of *PF* (0.84–1.29) values. However, the recommended model produced R^2 value of 0.60, λ^2 of 1.36, *AAE* of 9.80, and mean of *PF* of 1.00, which indicated that the suggested model showed the highest precision in determining the shear strength of *LS-SB-PC* beams.

The coefficients of ultimate compressive strength (*a*), tensile reinforcement ratio (*b*), span-depth ratio (*c*), fiber aspect ratio (*d*) and fiber volume (*e*) for the proposed model for shear strength prediction of the six groups of SFRC beams, as shown in Eqs. 7–12, are presented in Fig. 3. As

Table 6 Statistical analysis of shear strength prediction models for high strength SFRC beams with $a/d < 2.5$ and plain, crimped and hooked fibers (*HS-DB-PCH*).

Study	V_{EXP}/V_{ANA}				AAE	λ^2	R^2	Rating[b] (1–9)
	Mean	SD	COV (%)	Confidence[a]				
Ashour et al. (1992)	2.42	0.92	37.90	0.26	51.3	319.6	0.42	8
Imam et al. (1994)	0.63	0.31	49.78	0.09	122.2	328.4	0.40	9
Khuntia et al. (1999)	1.65	0.44	26.86	0.12	35.5	116.4	0.60	3
Kwak et al. (2002)	0.82	0.25	30.30	0.07	40.9	55.8	0.57	4
Li et al. (1992)	0.88	0.24	26.89	0.07	32.0	29.5	0.69	2
Narayanan and Darwish (1988)	0.99	0.34	33.78	0.09	35.7	63.1	0.48	5
Sharma (1986)	1.70	0.58	34.04	0.16	37.2	148.6	0.29	4
Shin et al. (1994)	1.07	0.33	31.36	0.09	30.4	33.6	0.51	9
Proposed Model	1.02	0.29	28.79	0.08	18.1	15.7	0.77	1

SM simplified model.
[a] $\alpha = 0.05$, [b] 1(excellent) and 9(worst).

Table 7 Statistical analysis of shear strength prediction for high strength SFRC beams with $a/d \geq 2.5$ and plain, crimped and hooked fibers (*HS-SB-PCH*).

Study	V_{EXP}/V_{ANA}				AAE	λ^2	R^2	Rating[b] (1–9)
	Mean	SD	COV (%)	Confidence[a]				
Ashour et al. (1992)	1.16	0.25	21.72	0.08	19.0	11.4	0.52	4
Imam et al. (1994)	1.15	0.23	19.66	0.07	17.8	7.1	0.54	3
Khuntia et al. (1999)	1.08	0.31	28.43	0.09	27.0	15.6	0.19	7
Kwak et al. (2002)	0.99	0.18	18.06	0.05	15.6	17.4	0.64	2
Li et al. (1992)	0.73	0.23	31.30	0.07	56.7	44.6	0.00	9
Narayanan and Darwish (1988)	1.01	0.25	24.63	0.08	21.7	17.0	0.38	6
Sharma (1986)	0.97	0.38	39.64	0.12	40.8	23.0	0.00	8
Shin et al. (1994)	1.04	0.22	20.94	0.07	17.6	11.8	0.44	5
Proposed model	0.99	0.14	14.61	0.04	14.3	10.0	0.66	1

[a] $\alpha = 0.05$, [b] 1(excellent) and 9(worst).

can be seen, the ultimate compressive strength had more influence on the shear strength of the *LS-DB-PC* beams, whereas a negative impact was observed for the low compressive strength SFRC beams containing hooked fibers (Fig. 3a). The shear capacity of the SFRC beams increased with an increase in the tensile reinforcement ratio, and it varied mainly on the beam configurations (Fig. 3b). Additionally, the shear strength of the SFRC beams varied

Table 8 Statistical analysis of shear strength prediction models for low strength SFRC beams with $a/d \geq 2.5$ and hooked fibers (*LS-SB-H*).

Study	V_{EXP}/V_{ANA}				AAE	λ^2	R^2	Rating[b] (1–8)
	Mean	SD	COV (%)	Confidence[a]				
Imam et al. (1994)	1.39	0.67	48.32	0.17	37.6	99.3	0.38	6
Khuntia et al. (1999)	1.68	1.03	61.18	0.26	33.3	203.4	0.24	5
Kwak et al. (2002)	1.24	0.68	54.27	0.17	25.6	97.2	0.42	4
Li et al. (1992)	1.11	0.70	62.61	0.18	37.8	92.5	0.10	7
Narayanan and Darwish (1988)	1.32	0.72	54.08	0.18	22.9	118.7	0.40	2
Sharma (1986)	1.39	1.00	71.81	0.26	35.1	163.4	0.05	3
Proposed model	1.34	1.78	133.34	0.46	46.3	153.5	0.54	1

[a] $\alpha = 0.05$, [b] 1(excellent) and 7(worst).

Table 9 Statistical analysis of shear strength prediction models for low strength SFRC beams with $a/d \geq 2.5$ and plain and crimped fibers (*LS-SB-PC*).

Study	V_{EXP}/V_{ANA}				AAE	λ^2	R^2	Rating[b] (1–9)
	Mean	SD	COV (%)	Confidence[a]				
Greenough and Nehdi (2008)	1.29	0.25	19.58	0.09	174.3	51.0	0.36	8
Imam et al. (1994)	1.15	0.27	23.13	0.09	22.6	5.8	0.19	7
Khuntia et al. (1999)	1.28	0.25	19.43	0.09	21.1	8.3	0.43	4
Kwak et al. (2002).	1.02	0.24	23.95	0.09	18.5	4.4	0.24	6
Li et al. (1992)	0.84	0.13	15.91	0.05	23.7	4.0	0.33	2
Narayanan and Darwish (1988)	1.06	0.24	22.17	0.08	17.6	4.2	0.39	3
Sharma (1986)	1.15	0.22	18.82	0.08	18.6	5.2	0.12	5
Proposed model	1.00	0.12	12.28	0.04	9.8	1.4	0.60	1

[a] $\alpha = 0.05$, [b] 1(excellent) and 8(worst).

depending on the beam geometry (span-depth ratio). As can be seen from Fig. 3c, the SFRC beams having a/d ratios of less than 2.5 had no significant effect on its shear capacity.

The study also revealed the identical conclusion conducted by Madan et al. (2007), the average shear stress of the SFRC beams decreases with increasing beam depth. The fiber-aspect ratio of the SFRC beams was also an influencing parameter on the shear capacity (Fig. 3d). Regardless of the shape and the aspect ratio of the steel fibers, the shear capacity of the SFRC beams was significantly affected by the volume of steel fibers (Fig. 3e). It was shown that the beams having a/d ratios of less than 2.5 contributed more resistant to shear capacity than those having a/d ratios of greater than or equal to 2.5. The results also indicated that the beams prepared with steel hooked fibers had more resistance to shear forces than those made with the plain and crimped steel fibers.

4.2 Principal Component Regression (PCR) Analyses

4.2.1 PCR Model on Entire Dataset

The principal component analysis (PCA) was conducted with all independent variables of compressive strength (CS), reinforcement ratio (RR), span-depth ratio (SDR), aspect

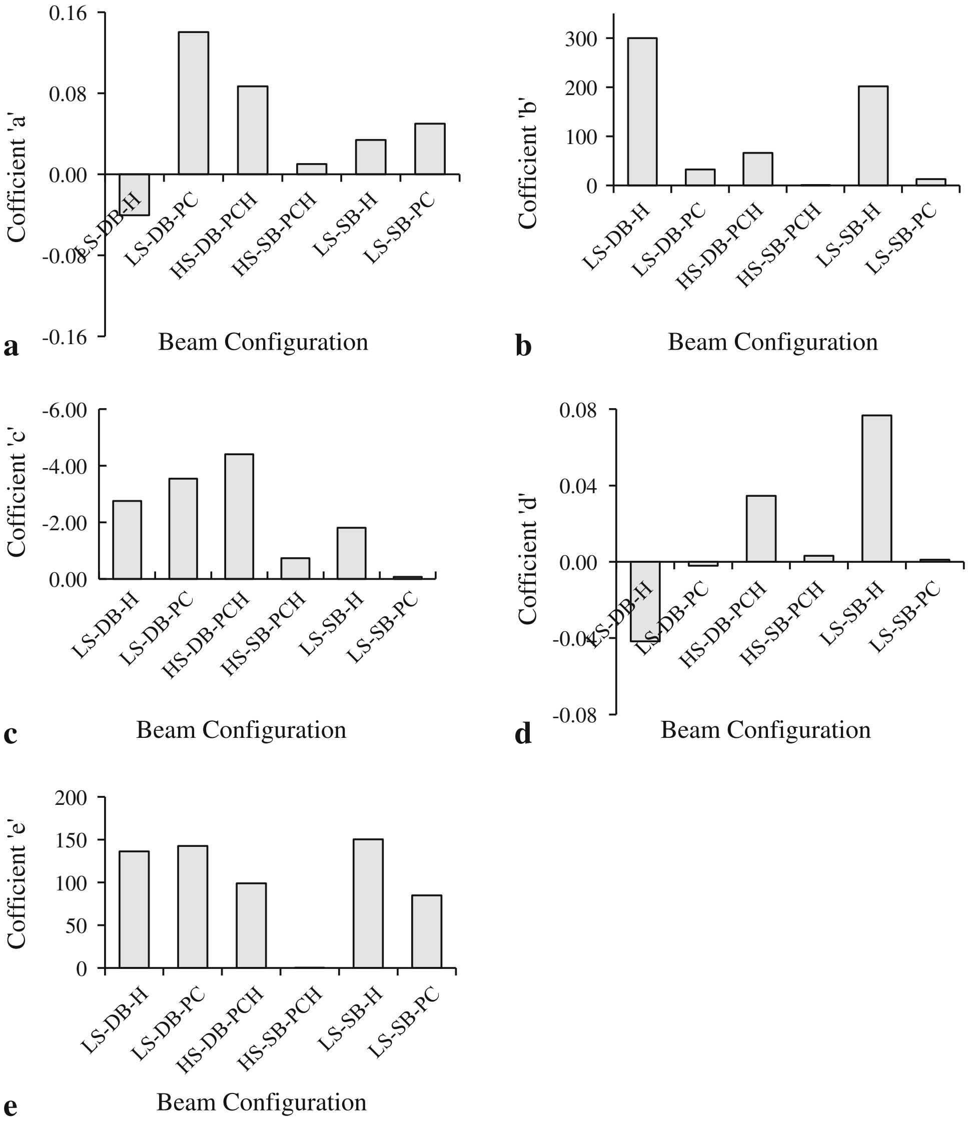

Fig. 3 Coefficients of factor influencing the shear capacity versus SFRC beam types.

ratio (AR), fiber volume (FV), and fiber type (FT). The result showed that the first four principal components (PCs) explained a total of 94 % variability of dataset. A biplot of the first principal component (PC1) versus the second principal component (PC2) is shown in Fig. 4, where the samples were displayed as points, and the variables were displayed as a vector. Moreover, the variables contributed similar information was grouped together. As can be shown from Fig. 4, the compressive strength (CS) and fiber type (FT) showed similar information and they were positively correlated. The two variables of the reinforcement ratio (RR) and fiber volume (FV) also expressed identical information. The distance to the vector from the origin also conveyed important evidence; farther away from the origin showed the strongest influence on the model. As such, the aspect ratio (AR) had less impact on the model as compared to the influence on the model by the remaining five variables (CS, FT, RR, FV and SDR).

The principal component regression analysis was performed on the shear strength and the scores of the first four principal components. The model is presented in Eq. (13). The predicted shear strength of all SFRC beams, evaluated by the PCR model of Eq. (13), was correlated with the observed shear capacity of the respective beams, and the relationship is presented in Fig. 5. As can be shown, a very good correlation with an R^2 value of 82.2 % was observed.

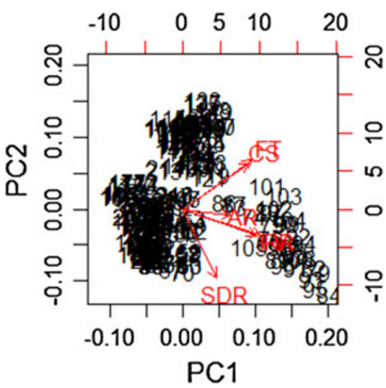

Fig. 4 Biplot of first two principal components of the total dataset.

$$SS = -4.224 + 0.053\ s1 + 1.350\ s2 + 0.0561\ s3 - 0.605\ s4 \quad (13)$$

where SS is the shear strength of SFRC beams, and s1, s2, s3 and s4 are the scores of the first, second, third and fourth principal components, respectively

4.2.2 PCR Model on Various Groups of SFRC Beams

In this analysis, the entire dataset was divided into six groups based on the ultimate compressive strength, span-depth ratio and the shape of steel fibers used in the beams, as shown in Table 3. For each group of SFRC beams, the principal component analysis was conducted on the five variables of compressive strength (CS), reinforcement ratio (RR), span-depth ratio (SDR), aspect ratio (AR) and fiber volume (FV). It was shown that the first four principal components explained 90.77, 94.68, 92.90, 92.50, 92.14 and 97.15 % variations in the dataset of LS-DB-H, LS-DB-PC, LS-DB-PC, HS-SB-PCH, LS-DB-PC and LS-SB-PCH beams, respectively.

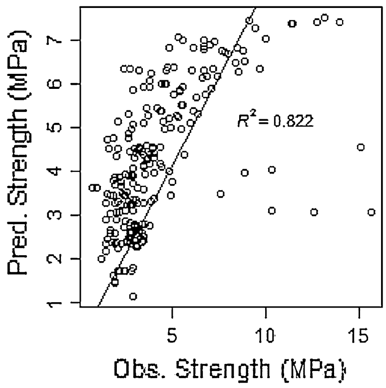

Fig. 5 The observed and predicted shear strength of all SFRC beams.

The biplot of the first two principal components for each SFRC beam group is shown in Fig. 6. As can be shown, a wide range of correlations was existed among the five variables. The relationships among the variables depended mostly on the configurations of SFRC beams. For instance, for the LS-DB-H beams, the RR, FV and AR variables expressed similar information and they were closely correlated, while the relationships among these variables were wide for the LS-DB-PC and LS-SB-PC beams. Additionally, fiber volume (FV) contributed positive impact on the LS-DB-PC beams, whereas it's influence was negative on the HS-DB-PCH, LS-SB-H and LS-SB-PC beams. The FV had most influence on the HS-SB-PCH beam, and on the other hand, it's contribution was minor on the HS-DB-PCH beams. The FV and AR variables were negatively correlated on the LS-DB-PC beams, and they were positively associated on the LS-DB-H beams. It can be demonstrated that each variable contributed differently on the SFRC beams depending on the ultimate compressive strength, span-depth ratio and shape of the fiber used in the beams.

For each type of SFRC beams, the principal component regression analysis was performed on the shear strength and the scores of the first two principal components.

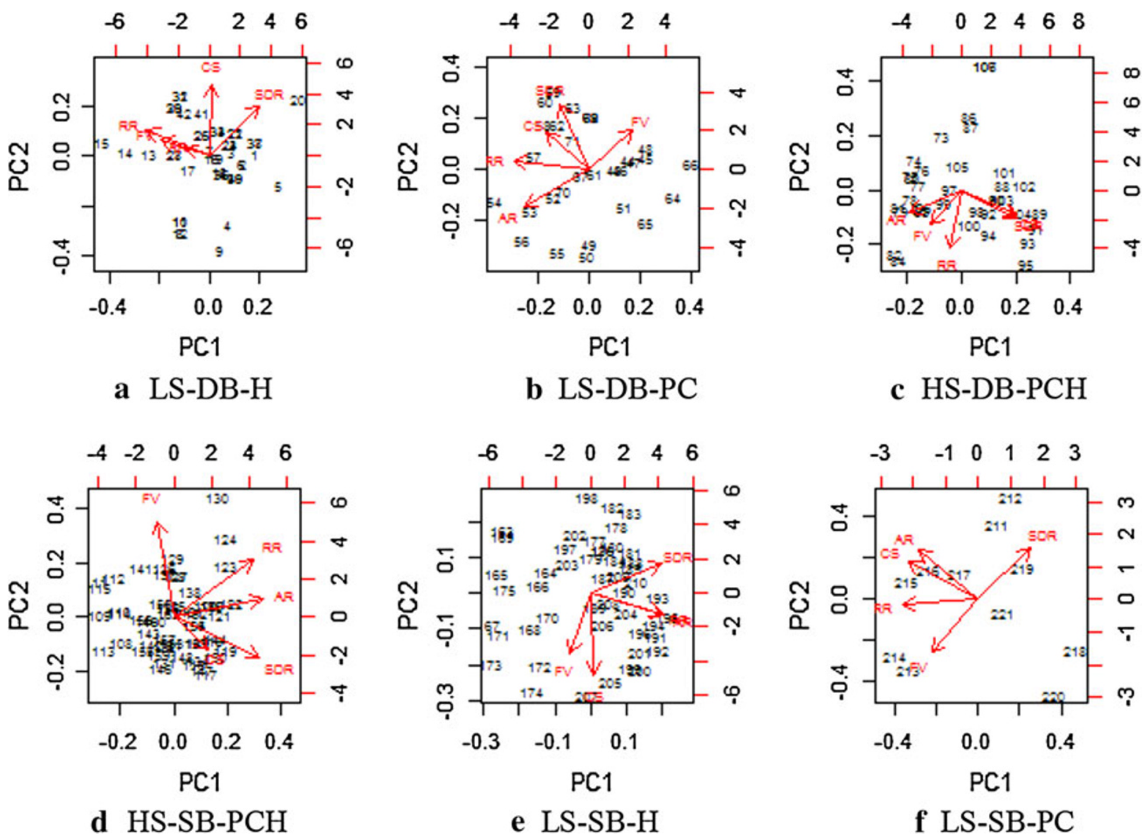

Fig. 6 The bi-plots of PC1 and PC2 for each group of SFRC beams.

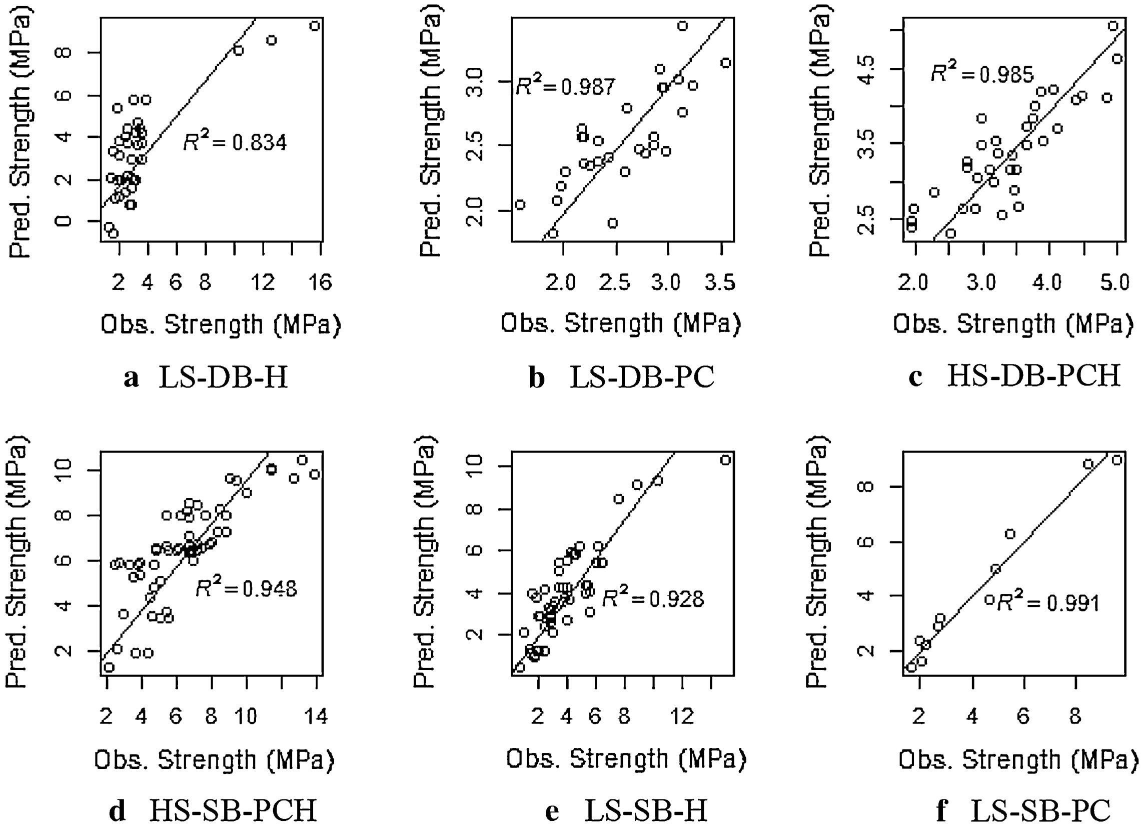

Fig. 7 Comparison of the observed shear strength versus that obtained by the principal component regression.

Equations (14)–(19) show the model in predicting the shear capacity of the LS-DB-H, LS-DB-PC, LS-DB-PC, HS-SB-PCH, LS-DB-PC and LS-SB-PCH beams, respectively. The predicted shear strengths of various types of SFRC beams was evaluated using the PCR models of Eqs. 14–19, and they was compared with the observed shear capacity of the corresponding beams. The results are shown in Fig. 7. As can be shown, a good correlation existed with R^2 values of 0.834, 0.987, 0.985, 0.945, 0.928 and 0.991 between the predicted shear strength and that obtained by the experimental procedures for the *LS-DB-H*, *LS-DB-PC*, *HS-DB-PCH*, *HS-SB-PCH*, *LS-SB-H*, and *LS-SB-PC* beams, respectively.

$$SS = 3.395 - 1.668\ s1 + 0.239\ s2 + 0.482\ s3 - 0.336\ s4 \tag{14}$$

$$SS = 2.539 - 0.139\ s1 + 0.178\ s2 + 0.280\ s3 + 0.061\ s4 \tag{15}$$

$$SS = 3.381 - 0.405\ s1 - 0.194\ s2 - 0.004\ s3 + 0.350\ s4 \tag{16}$$

$$SS = 6.478 - 0.143\ s1 + 1.140\ s2 + 0.889\ s3 - 2.072\ s4 \tag{17}$$

$$SS = 4.036 - 0.188\ s1 - 0.754\ s2 - 0.798\ s3 + 2.490\ s4 \tag{18}$$

$$SS = 4.230 - 1.530\ s1 - 0.870\ s2 + 0.720\ s3 - 0.068\ s4 \tag{19}$$

where SS is the shear strength of SFRC beams, and s1, s2, s3 and s4 are the scores of the first, second, third and fourth principal components, respectively

5. Conclusions

The outcomes of this research study can be summarized as follows:

1. The shear capacity of the SFRC beams increased with an increase in the tensile reinforcement ratio, the shear span-depth ratio of the beam and the ultimate compressive strength.
2. Regardless of the shape and the aspect ratio of the steel fibers, the shear capacity of SFRC beams increased with an increase in the fiber volume with an exception of high strength beams made with plain, crimped and hooked fibers and having span-depth ratios of more than or equal to 2.5.
3. The shear strength of the SFRC beams predicted by the previously suggested models was typically in excess of or smaller than the experimental results. Of all the suggested existing models, the model suggested by Narayanan and Darwish (1988) was the most accurate for the shear strength prediction of the *LS-DB-H* and

LS-SB-H beams, and that recommended by Li et al. (1992) for the *HS-DB-PCH* and *LS-SB-PC* beams, respectively. The empirical equation proposed by Kwak et al. (2002) for the *LS-DB-PC* beams, and the model recommended by Khuntia et al. (1999) for the *HS-SB-PCH* beams, respectively. The study also showed that the model suggested by Sharma (1986), which is currently being used by the ACI, was the most inaccurate in generating the shear strength of SFRC beams. The study showed that the predictions made by all existing models varied quite significantly and indicated inaccurate when they were compared with a large experimental database.

4. The study indicated that the theoretical shear strength of SFRC beams computed by the suggested model were more precise and more accurate compared to those evaluated by the existing shear strength prediction models.
5. Principal component regression models outperformed not only the existing equations but also the multivariate regression model presented herein in evaluating the shear strength of the SFRC beams.

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